Advances in the use of simplicial Finite Elements for Flow problems

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The simulation of problems in CFD requires addressing the discretization of increasingly complicated objects posing increasing modelling challenges. Despite the advances of the meshing technology, the generation of suitable meshes around complex geometrical objects still represents an overwhelming challenge, particularly when the use of hexahedra elements is required.

This fact motivated the rise of alternative simulation approaches (Lattice Boltzmann, Finite Differences, Finite Element Embedded approaches), which approximate the shape of the object of interest by the use of local refinement techniques or simply by employing finer discretizations. Within the area of the Finite Element Method (FEM), the challenge is often addressed by the use of tetrahedral elements which naturally lead to the definition of boundary fitted meshes, when triangulated surfaces are used as starting point. The use of simplex elements makes also available mesh refinement techniques also in the context of the FEM \cite{1}. Mixed velocity-pressure elements, stabilized using VMS-type or FIC stabilizations \cite{2}\cite{3} are employed to form the computational skeleton of the CFD solver developed.

One of the goals of current paper is to describe recent advances in our Finite Element technology in embedding information of large obstacles to be included in flow computations. The message here is that the use of tetrahedral elements greatly simplifies geometrical operations and hence allows performing efficiently local enrichment operations, which in turn allow the use of suitable enriched spaces and hence satisfactory approximation properties. This characteristics, together with the ability of robustly refining the grid as needed, make simplicial elements very suitable for embedded computations.

Our objective will be here to describe how existing velocity-pressure elements can be retrofitted to incorporate in the space the essential features needed for the flow computations around embedded objects. This includes for example adding discontinuities in the mixed fields and/or enriching the gradient of the different fields.

Our second goal is to show how the accuracy of existing u-p elements could be improved by the introduction of a third unknown field, without changing the geometrical discretization, that is, still using tetrahedral elements. We will show how such improved technique allows an efficient semi-explicit solution scheme while preserving favorable properties for the stable time step.

Finally we will discuss how all of the proposed modifications retain very convenient parallelization properties, which make them suitable for very large scale computations.

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